AD-A275 338

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1

1. AGENCY USE ONLY (*Leave blank*)	2. REPORT DATE Dec 93	3. REPORT TYPE AND DATES COVERED THESIS/~~DISSERTATION~~

4. TITLE AND SUBTITLE
The Appearance of the Sun and Moon Seen through Clounds

5. FUNDING NUMBERS

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
AFIT Student Attending: Penn State Univ

8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT/CI/CIA- 93-147

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
DEPARTMENT OF THE AIR FORCE
AFIT/CI
2950 P STREET
WRIGHT-PATTERSON AFB OH 45433-7765

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

DTIC
S ELECTE D
FEB 4 1994
C

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release IAW 190-1
Distribution Unlimited
MICHAEL M. BRICKER, SMSgt, USAF
Chief Administration

12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*)

66P8

94-03910

14. SUBJECT TERMS

15. NUMBER OF PAGES
56

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT

NSN 7540-01-280-5500

94 2 03 127

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

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The Pennsylvania State University

The Graduate School

Department of Meteorology

THE APPEARANCE OF THE SUN AND MOON SEEN THROUGH CLOUDS

A Thesis in

Meteorology

by

Jeffery Robert Linskens

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

December 1993

We approve the thesis of Jeffery Robert Linskens.

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ABSTRACT

The sun occasionally appears fuzzy through altostratus because altostratus is composed of larger particles than other clouds, and is of the necessary optical thickness. Experimental results indicate that the range of optical thicknesses of a cloud at which a fuzzy sun is seen increases with the size of the particles. This relationship is caused by an increase in the attenuation of contrast at high spatial frequencies relative to that at low spatial frequencies when the size of cloud particles increases. The increase in the size of cloud particles is caused by the presence of raindrops and crystals in the cloud.

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ACKNOWLEDGEMENTS

I am grateful to Dr. Craig F. Bohren for suggesting this topic for my research, and for guiding me to the completion of my thesis. Due to his willingness to fund my attendance at relevant conferences, I received valuable insight on my research from David Lynch, William Livingston, and others whose names I do not recall. I must also thank Dr. Bohren for the photographs of the experiment that appear in this thesis; I have neither the equipment nor the skill to have taken them myself. His enthusiasm for the subject and patience with me made working under his tutelage a pleasure.

I also appreciate the interest Dr. Alistair B. Fraser has consistently shown in my work and his willingness to trust me with his *International Cloud Atlas* for several months.

Michael Churma improved the quality of my thesis by suggesting that we use his spectral radiometer to estimate optical thicknesses during the experiment. I am thankful to him for his contributions during the experiment, and to Duke Scientific Corporation for the particles that were used in the experiment.

This research was supported in part by National Science Foundation Grant ATM-8810876. Finally, I would like to thank the United States Air Force for giving me the opportunity to pursue a master's degree in meteorology at Penn State.

Chapter 1

INTRODUCTION

It has long been noticed that although the sun seen through clouds generally has a sharp edge, it occasionally appears fuzzy. At certain times, regardless of the contrast between the sun and the cloud, the edge of the sun cannot be identified. The fuzzy sun has long been associated with altostratus, but the reason for this association has never been explained. In this thesis I attempt to give an explanation by appealing to a combination of observations, experiments, and theory, each of which is discussed in turn. A manuscript based on this thesis has been submitted to *Applied Optics* for publication. Another manuscript is being prepared for submission to *Journal of the Atmospheric Sciences*.

My attempt to simulate the appearance of the sun through clouds by using Monte Carlo techniques to model multiple scattering of sunlight by clouds is described in the appendix. A copy of the computer code is included at the end of the appendix.

Chapter 2

OBSERVATIONS OF THE SUN AND MOON SEEN THROUGH CLOUDS

That the sun seen through clouds occasionally appears fuzzy has been documented in a few books. The association of the fuzzy sun with altostratus was published as early as 1934 by Ralph Abercromby in *Weather*.[1] It is stated in the *International Cloud Atlas* that one of the distinguishing features of altostratus is that it "prevents objects on the ground from casting shadows and that it may show a ground glass effect."[2] In fact, a fuzzy image of the sun can be seen in six of the eight photographs of altostratus in the atlas, but a fuzzy image of the sun is not seen in any of the hundreds of photographs of other cloud types.[3] It is suggested in the atlas that a fuzzy image of the sun can be used to distinguish altostratus from cirrostratus and stratus: the cloud should be classified as altostratus if the sun appears fuzzy through it. It is not clearly stated that the cloud should not be classified as altostratus if the sun is not fuzzy.[2] And van de Hulst devotes a section of *Multiple Light Scattering* to what he calls the hazy sun. He notes that Minnaert, among others, observed the phenomenon of the fuzzy sun, but remained puzzled about its cause. Van de Hulst's

concluding sentence on the fuzzy sun is "Further studies are necessary."[4]

Although the sun is frequently obscured by clouds, it is quite commonly visible through clouds. When the sun is visible, its limb generally appears sharp, as shown in Figure 2.1. Incidentally, no photograph will show the exact image an observer saw; photographs have been included only to illustrate what is being described. At other times, the sun is visible, yet its limb is not sharp. Figure 2.2 is a photograph of a fuzzy sun. The limb is fuzzy and the edge of the sun cannot be identified. Although sharp-edged sun are more common than fuzzy suns, fuzzy suns are not rare. I have

Figure 2.1. Sharp-edged sun viewed through patchy stratus.

observed numerous sharp-edged and fuzzy suns in the two years I have spent carefully observing the sun through clouds.

The unattenuated sun is sharp-edged, but frequently the sharp edge can be observed only with difficulty. As shown in Figure 2.3, the luminance pattern is approximately constant across the angular radius of the sun. At the edge of the sun, known as the solar limb, the luminance drops to approximately one tenth of one percent of the luminance within the solar disk. Beyond the solar limb, in the aureole, the luminance decreases gradually with angular distance from the center of the sun.[4] It is difficult to see the solar limb when the

Figure 2.2. Fuzzy sun viewed through altostratus.

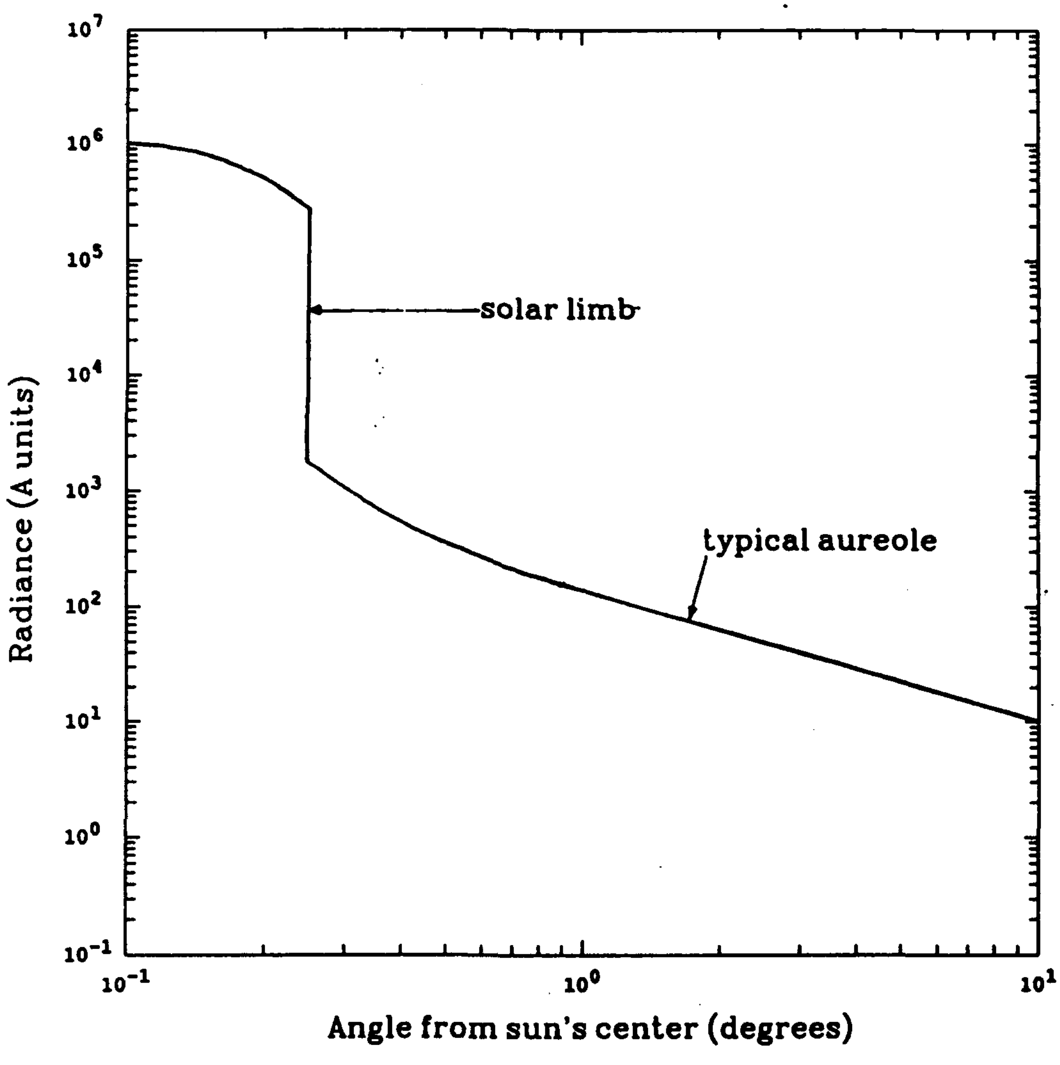

Figure 2.3. Radiance of the sun's disk and aureole under typical conditions. 1 A unit is 2.8 x 10^{-14} erg^{-3} sec^{-1} sr^{-1}. (After van de Hulst.[4])

slightly attenuated sun is observed in the overhead sky because the luminance of the disk and aureole are usually too great for the eye to distinguish one from the other. But it is not uncommon to look at the sun near the horizon, and, because of sufficient attenuation, observe the characteristic sharp limb.

Careful observation is necessary to prevent confusing an insufficiently attenuated sun and aureole with a fuzzy sun. The aureole results from single scattering that is peaked in the forward direction due to the size of the atmospheric particles. The aureole is much less bright than the solar disk, and, because the aureole is caused by single scattering, it exists when the medium through which the sun is observed is thin. The aureole can be attenuated and the solar limb can be observed by using sunglasses or by looking at the reflection of the sun in a piece of dark glass, such as the one shown in Figure 2.4. But the fuzzy sun is different from the aureole. Occurring at greater optical thicknesses than the aureole, it is the product of multiple scattering, not single scattering.[4] Using sunglasses to reduce the luminance of a fuzzy sun does not reveal a solar limb.

Fog has produced the most remarkable sharp-edged suns that I have seen. Even when the sun was greatly attenuated by fog, I always observed a sharp edge. At times, I have seen the sun become extremely faint, then become not visible, and then become visible again. Although this happened slowly, I

never observed fuzziness even when the sun was just barely visible through fog.

Fog is not the only cloud through which I have seen sharp-edged suns. I have seen them through stratus, with characteristics similar to the sharp-edged suns seen through fog; I have seen them also through stratocumulus and cumulus. The sharp-edged suns seen through stratocumulus and cumulus were transitory. As stratocumulus or cumulus drifted across the sky, the sun would change rapidly from being unattenuated by clouds, to being greatly attenuated and sharp-edged, to not being visible at all when the intervening cloud was thick. I have seen sharp-edged suns through high clouds also, such as

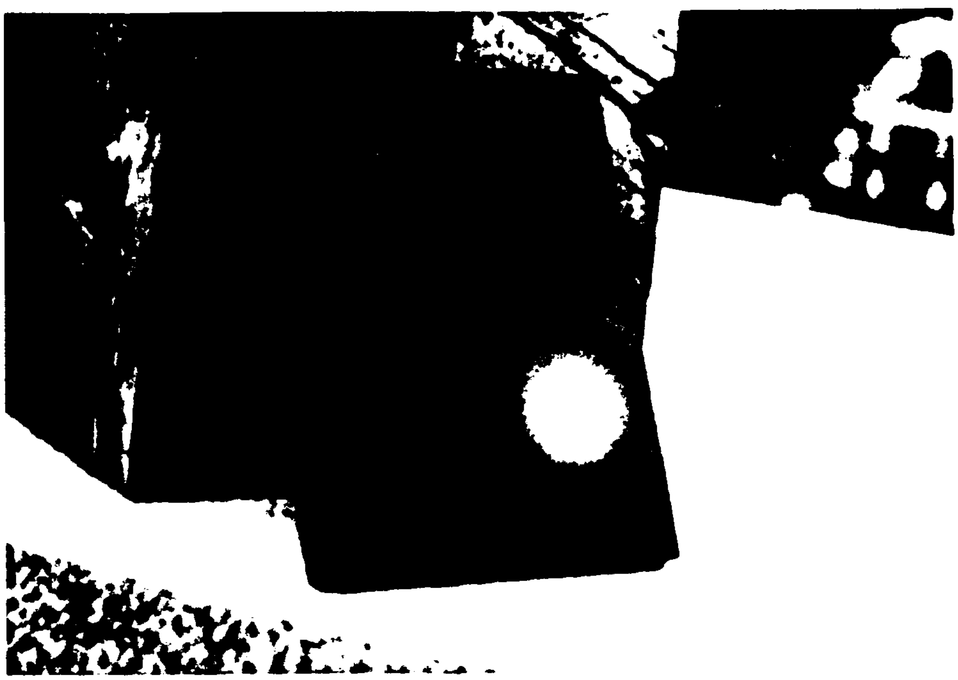

Figure 2.4. The use of dark glass to reduce the brightness of the sun. The sun reflected in the dark glass has a fuzzy edge.

cirrus, but the luminance was never so reduced that I could look at the sun and see the sharp edge without using sunglasses or reflecting glass.

Classifying clouds can be difficult, but when I have seen a fuzzy sun, I think it always has been through altostratus. I know it always has been through middle or high clouds, and never has been through low clouds. When I have watched the sky for hours at a time, I have observed a general pattern concerning fuzzy suns. First, the sky is virtually clear and the sun is too bright for direct observation. Then, wispy clouds that seem to be composed of ice crystals appear, and become thicker with time. The sun is still too bright to be looked at directly, but by using sunglasses or looking at the sun's reflection in a piece of dark glass I can reduce the luminance enough to see the sharp edge of the sun. The cloud continues to become thicker and the sun, which is still too bright to be looked at directly, appears fuzzy. The cloud becomes even thicker; the sun becomes more fuzzy, and dim enough to be looked at directly. Eventually, the sun is dimmer and fuzzier, and then it is not visible at all. It is not uncommon for rain to begin several hours later. I have seen fuzzy suns in all seasons, but they are most common in winter and least common in summer.

Only once have I observed the transition from a sharp-edged sun to a fuzzy sun while the transition was occurring. I was looking at the reflection of the sun through a cloud in

black glass while wearing sunglasses. The edge of the sun was sharp. Then I noticed very slight fuzziness. Confused, I put on a second set of sunglasses. Initially the sun appeared dimmer and sharper, then it became more fuzzy. Then the sun gradually became dimmer and even more fuzzy. After some time, I was able to look directly at the sun, which was fuzzy, without using sunglasses. Later, clouds obscured the sun.

I have observed not only the sun through clouds, but the moon as well. In fact, it is easier to observe the moon through clouds because it is much dimmer than the sun. The moon is never too bright to be looked at directly, and its aureole is distinguishable from its direct image. My comments concerning the appearance of the sun through clouds are true for the moon as well.

What can be learned by observing the sun through clouds is limited by their variability and by the continuous changes they undergo. Fundamental characteristics of clouds such as their structure, composition, drop size distribution, and thickness can neither be known precisely nor be controlled. I therefore performed a series of controlled experiments to learn more about the appearance of the sun through clouds.

Chapter 3

EXPERIMENTS

An experiment was conducted to investigate the relationship between both the size of cloud droplets and the optical thickness of the cloud through which the sun is viewed, and the sharpness of its image. A 60-watt light bulb and a fish tank (26 cm high, 26 cm wide, 50 cm long) filled with particles of known size suspended in distilled water were arranged in a dark room to simulate the sun seen through clouds. The light bulb was positioned relative to the observer so that it looked like a uniformly bright disk. The distance between the light bulb and the tank was such that the illumination of the tank was approximately uniform. The angular width of the light bulb as viewed by the observer was equal to the angular width of the sun when viewed from Earth ($\sim 0.5°$).

Haze, fog, and clouds were simulated by suspending three sizes of polystyrene spheres (provided by Duke Scientific Corporation) in distilled water in the fish tank. Haze droplets were represented by particles with mean diameter of 0.652 µm (standard deviation, 0.0048 µm). Fog droplets were represented by particles with mean diameter of 5.3 µm

(standard deviation, 1.2 μm). Cloud droplets were represented by particles with mean diameter of 15.8 μm (standard deviation, 2.9 μm). Their indices of refraction are 1.59 at 589 nm (0.652 μm diameter particles) and 1.59 at 540 nm (other particles). They are virtually non-absorbing at visible wavelengths.[5]

The particles, which were packaged as aqueous suspensions at 10% solids, were added in small increments to the distilled water using an eye dropper. After drops were added, the water in the tank was stirred to make the distribution of particles uniform and then was allowed to become still to minimize turbulence. Stirring sometimes caused air bubbles to form on the glass walls of the tank; bubbles were removed after the water came to rest. No two eyes see alike, so three observers viewed the light bulb through the suspensions. The results reported here represent the consensus of the observers on what they saw.

The optical thickness of the suspensions was estimated from the number of drops of the aqueous suspensions added to the distilled water using a SpectraScan PR-704 spectral radiometer. The spectral radiance of the inner half of the light bulb was measured with the two glass walls of the tank and 26 cm of distilled water as the intervening medium. The spectral radiance of the central half of the light bulb was measured at the same distance from the light bulb each time drops of fluid were added to the tank. Optical thicknesses

were estimated by plotting $-\ln(L/L_o)$ against the number of drops, where L is the spectral radiance measured through a given number of drops and L_o is the spectral radiance measured through only the tank and distilled water. The curve was extrapolated to zero drops and its slope at zero drops was used to estimate optical thickness as a function of the number of drops. Measurements were taken at 700 nm, long enough to minimize the effect of preferential scattering by the smallest particles, but shorter than an absorption band of water at slightly longer wavelengths. The optical thicknesses reported pertain to the suspended particles only.

As 0.652 μm particles were added to the distilled water, the edge of the light bulb remained sharp until its image could be seen only faintly. Reddening of the image due to preferential scattering of short wavelengths was apparent at optical thicknesses as low as 1.0. The sharp edge and the absence of an aureole are evident in Figure 3.1, which is a photograph of the light bulb through 0.652 μm particles with an optical thickness of about 8.4. The image of the light bulb still appeared to be a disk, but the edge began to appear fuzzy at an optical thickness of about 8.8. Not only did fuzziness increase as optical thickness increased beyond 8.8, but the shape of the image became less distinct as well. The image of the light bulb could not be seen when the optical thickness was about 9.4. The light bulb appeared fuzzy during approximately the greatest 7% of the optical thicknesses at

which it was visible. In this thesis, when the sun or the light bulb are said to be visible, it is meant that a bright spot caused by one of them, however faint or indistinct, can be distinguished from the background.

As 5.3 µm particles were added to distilled water, the edge of the light bulb remained sharp until the image of the light bulb was faint. At an optical thickness of about 1.2 an aureole and a corona with an inner radius of about 1.5° were visible. The corona became less pronounced as particles were added to the water; the corona was not observed at an optical thickness of about 7.5. The edge of the light bulb was sharp

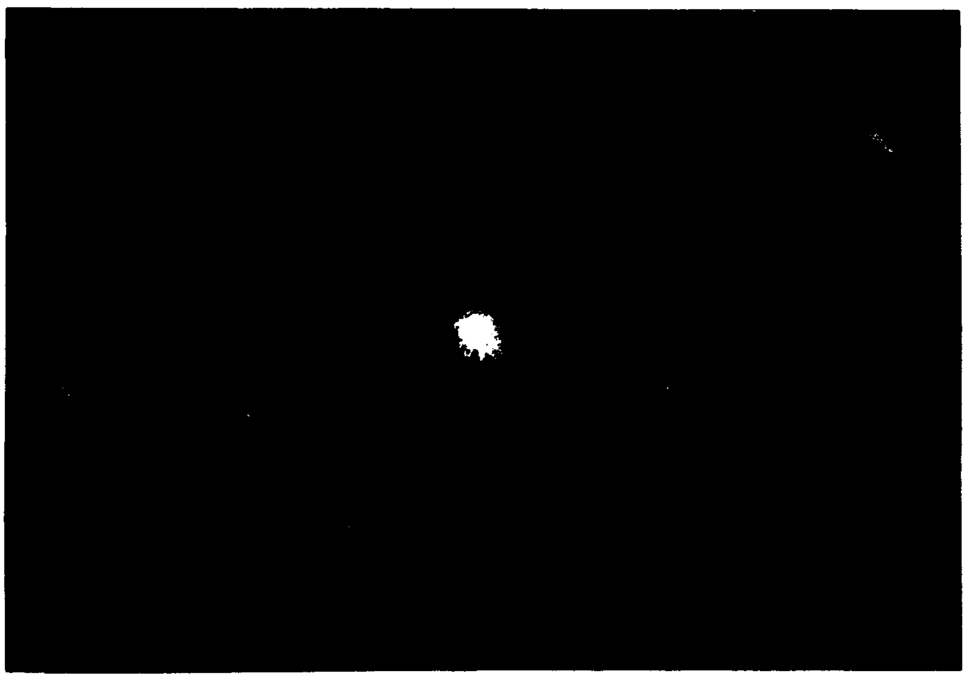

Figure 3.1. Light bulb viewed through 0.652 µm particles with an optical thickness of about 8.4. Notice the sharp edge of the light bulb and the absence of an aureole.

and distinct from the aureole until the optical thickness was about 10.9. Figure 3.2 is a photograph of the light bulb through 5.3 µm particles at an optical thickness of about 6.3: the sharp edge of the light bulb is evident. The edge of the light bulb could not be seen beyond an optical thickness of 10.9; the light bulb appeared fuzzy. Not only did fuzziness increase as optical thickness increased beyond 10.9, but the shape of the image became less distinct as well. The light bulb ceased to be visible when the optical thickness was about 11.8. The light bulb appeared fuzzy during approximately the

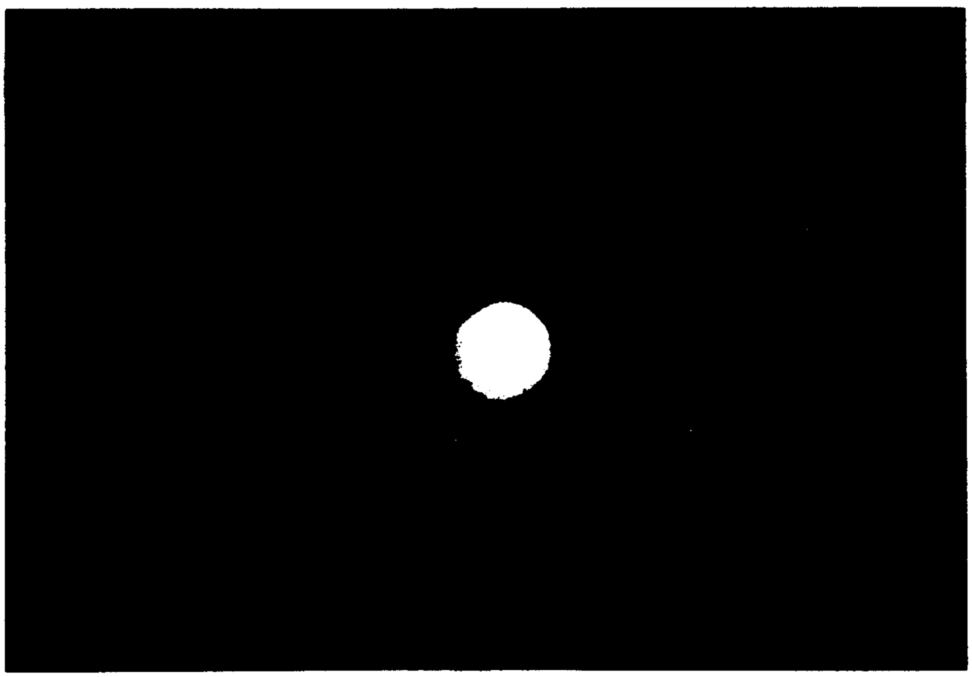

Figure 3.2. Light bulb viewed through 5.3 µm particles with and optical thickness of about 6.3. Notice the sharp edge of the light bulb. The surrounding aureole is less evident in the photograph than it was to the observers.

greatest 8% of the optical thicknesses at which it was visible.

As 15.9 μm diameter particles were added to the distilled water, the edge of the light bulb became fuzzy at a smaller optical thickness than it did with the two smaller particles. The addition of only a few particles to the distilled water produced an aureole and a dramatic corona. The corona, which had a smaller angular radius than the corona associated with the 5.9 μm particles, remained visible until an optical thickness of about 5.2. The edge of the light bulb could not be seen beyond an optical thickness of 9.8; the light bulb appeared fuzzy. Not only did fuzziness increase as optical thickness increased beyond 9.8, but the shape of the image became less distinct as well. Figure 3.3 is a photograph of the light bulb through 15.9 μm particles at an optical thickness of about 10.3: the sharp edge of the light bulb is not evident. The light bulb ceased to be visible when the optical thickness was about 12.8. The light bulb appeared fuzzy during approximately the greatest 23% of the optical thicknesses at which it was visible.

Several general conclusions can be drawn from the results of the experiment. For a given particle size, there is a range of optical thicknesses for which the sun's edge is distinct, a range of optical thicknesses for which the edge is fuzzy, and a range of optical thicknesses for which the sun cannot be seen at all. The second of these ranges increases

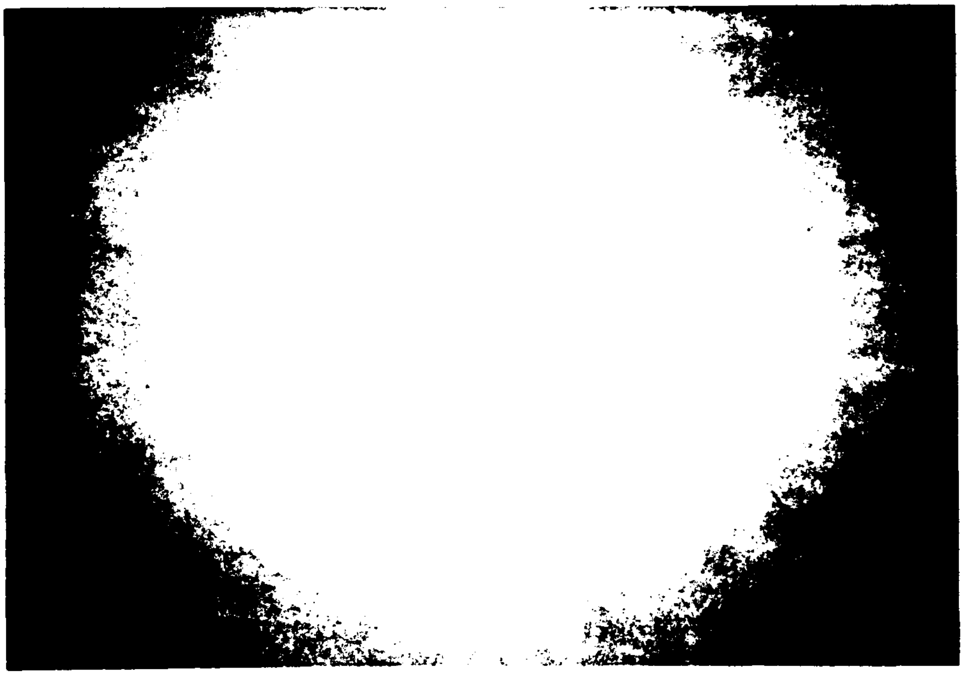

Figure 3.3. Light bulb viewed t. rough 15.9 μm particles with an optical thickness of about 10.3. Notice the fuzzy edge of the light bulb.

with increasing particle size. The fuzziness observed through the two smallest particles was observed at extremely low contrasts; the light bulbs were seen because the observers were looking carefully as the optical thickness of the particles changed very gradually. With the smallest two particles, a casual observer might not have noticed the fuzziness before the light bulb ceased to be visible. Also, if the image of the light bulb was fuzzy, the fuzziness increased as optical thickness increased.

Chapter 4

OTHER POSSIBLE CAUSES OF THE FUZZY SUN

This paper is primarily about the relationship between the size of cloud droplets and the appearance of the sun as seen through clouds. Other factors, such as the shape of ice crystals, turbulence, and horizontal inhomogeneities of clouds, are sometimes suggested as causes of the fuzzy sun. These possibilities are not disproved, but the results of the experiment indicate that they are not necessary for a fuzzy sun to be seen.

The non-sphericity of ice crystals is sometimes suggested as a possible cause of the fuzzy sun because altostratus is generally, but not always, partially composed of ice crystals.[2] But van de Hulst has found that randomly oriented cylinders form a near-forward scattering pattern strikingly similar to that of spheres.[4] It would be remarkable, therefore, if the shape of ice crystals in clouds were the cause of the fuzzy sun.

Turbulence may degrade the quality of images because of variations in the refractive index due to temperature inhomogeneities.[6] This has been observed while looking at the sun through a plume from a smoke stack. The temperature

variations necessary for turbulence would be in the horizontal, not the vertical dimension because the visual path is roughly vertical. Such temperature variations are small. Also, the experiment indicates that fuzzy suns can be observed in the absence of turbulence.

Horizontal inhomogeneities in clouds could cause a fuzzy sun. It is not uncommon to observe half the sun while the other half is obscured by a cloud. If inhomogeneities in the optical thickness of a cloud were great enough to make some sections of the sun visible, and to make other sections not visible, and if the inhomogeneities were on a horizontal scale approximately equal to the smallest angular distance that can be resolved by the human eye, the sun may appear fuzzy. But altostratus is a fairly uniform cloud formed by the slow ascent of extensive layers of air. Several other cloud types, cumulus, stratocumulus, and altocumulus in particular, are less uniform horizontally than altostratus.[2] Fuzzy suns are not seen through these clouds. Also, the experiment indicates that fuzzy suns can be observed through homogeneous media.

Chapter 5

THE EYE

How the eye sees is relevant to how an intervening medium, such as a cloud, degrades image resolution. The eye resolves images, such as the sun as seen through a cloud, in a manner remarkably similar to Fourier analysis. Any spatial pattern can be broken down into a set of sine waves of various spatial frequencies, which can be summed to produce the original pattern. For example, a square wave of frequency f and amplitude 1 can be analyzed into the sum of sine waves with frequencies that are odd integer multiples of f. The square wave can then be approximated by

$$4/\pi\ [\sin(f) + (1/3)\sin(3f) + \ldots + (1/n)\sin(nf)]. \qquad (5.1)$$

As can be seen in Figure 5.1, the extent to which the square wave is approximated by its Fourier analysis is a function of n. As n increases and higher frequencies are included, the edges of the pattern become sharper and the approximation of the pattern becomes closer to the original square wave.[7]

One important difference between how the eye sees and how signals are Fourier analyzed is that the eye does not detect

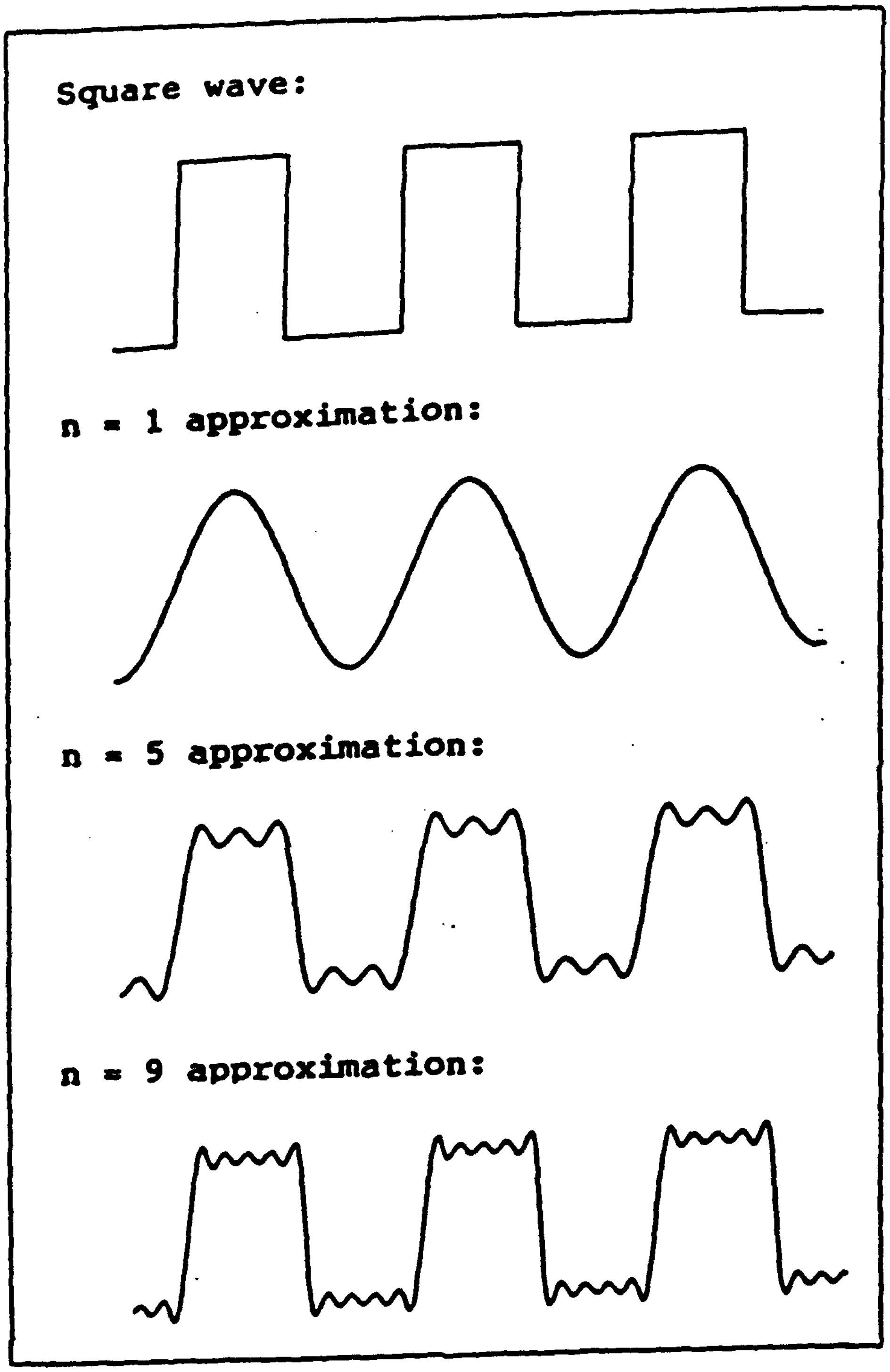

Figure 5.1. Fourier analysis of a square wave. The extent to which the Fourier analysis approximates the square wave is a function of the highest frequency included in the approximation.

the absolute value of a signal; it detects the contrast between the image of an object and the image of its background. A visual scene can be thought of as a complex waveform composed of sine waves of the appropriate frequencies and amplitudes. Two criteria must be met for an object, part of the visual scene, to be detected. First, the angular width of the object must be greater than the inverse of the highest spatial frequency that the eye can detect. Second, the contrast between the image of the object and that of its background must be greater than the threshold contrast. Even if the spatial frequency criterion is met, if the contrast between the object and the background at every spatial frequency is less than the threshold contrast, which is a function of spatial frequency, the image is not detectable.[8]

These criteria are summarized by the contrast sensitivity function (CSF) of the human eye, which is shown in Figure 5.2. The eye is sensitive to spatial frequencies between about 0.5 cycles per degree (c/deg) and 50 c/deg. The angular width of the sun is about 0.5° so the first criterion is met. Contrast threshold is the inverse of contrast sensitivity, which is the vertical axis in Figure 5.2. The eye is most sensitive to about 5 c/deg, where the contrast threshold is less than 1%. The sensitivity of the eye decreases sharply as the spatial frequency decreases. Sensitivity also decreases, but more gradually, as spatial frequency increases. At the highest spatial frequency the eye can detect, about 50 c/deg, the

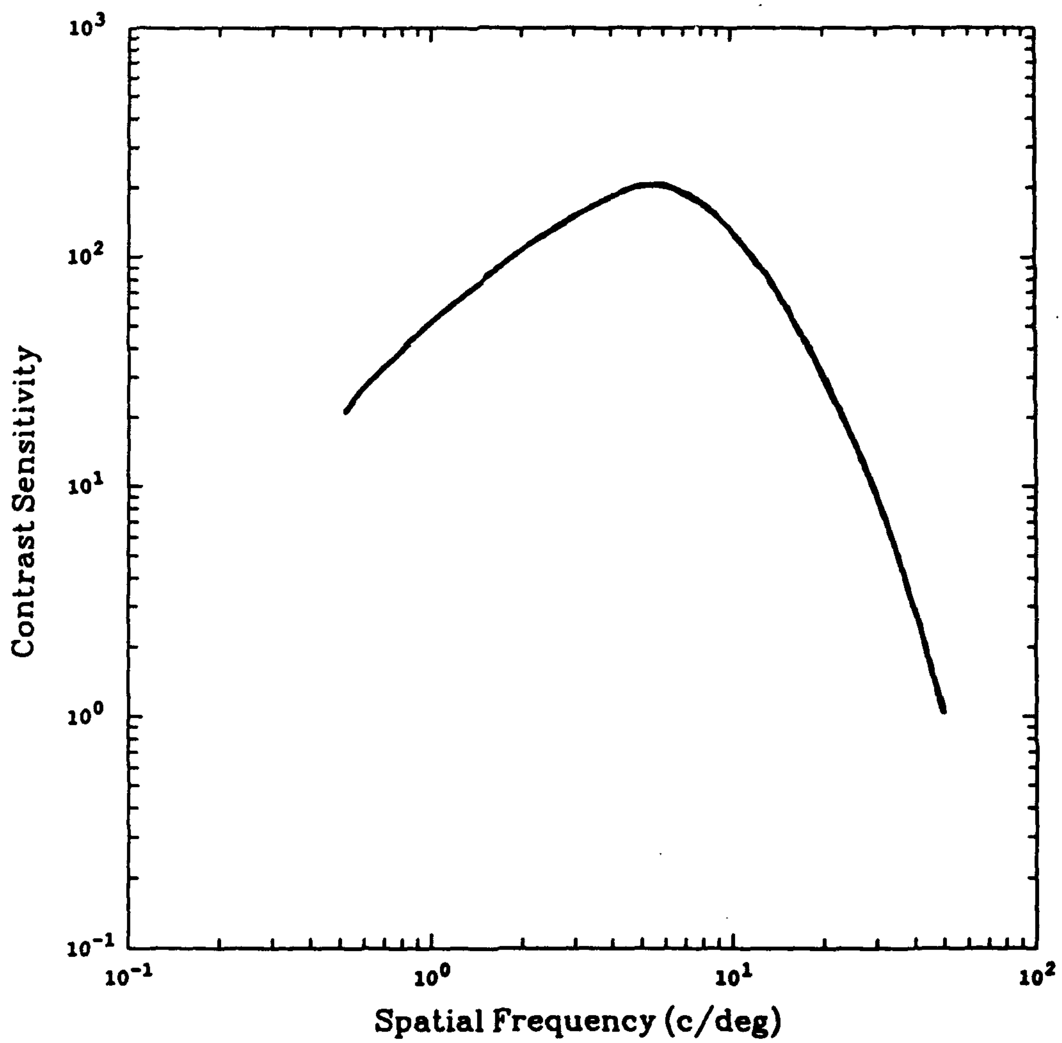

Figure 5.2. Contrast sensitivity function of the human eye. (After DeValois and DeValois.[8])

contrast threshold is about 100%.[9] The CSF depends on absolute luminance; the CSF in Figure 5.2 is for 5 Lamberts. As luminance decreases, the CSF shifts downward and its peak shifts to lower frequencies.[8]

The extent to which the sine wave components of the square wave approximate the square wave is determined by the highest frequency sine wave included in the approximation. Similarly, the extent to which the eye can resolve the sun is determined by the highest spatial frequency at which the contrast between the sun and the background cloud is greater than the contrast threshold. If the contrast between the sun and the background cloud at 50 c/deg is greater than the contrast threshold at 50 c/deg, the sun will appear at its sharpest. If the contrast between the sun and the background cloud is less than the contrast threshold at all spatial frequencies at which the eye is sensitive, the sun will not be seen. If the contrast between the sun and the background cloud is greater than the contrast threshold below an intermediate spatial frequency, but it is less than the contrast threshold at higher spatial frequencies, the sun will be visible, but with some degree of fuzziness. The lower the spatial frequency at which the contrast is less than the contrast threshold, the more fuzzy the sun will be.[8]

I observed the gradual degradation in sharpness that is described in the preceding paragraph while performing the experiment. Also, if the light bulb appeared fuzzy when I was

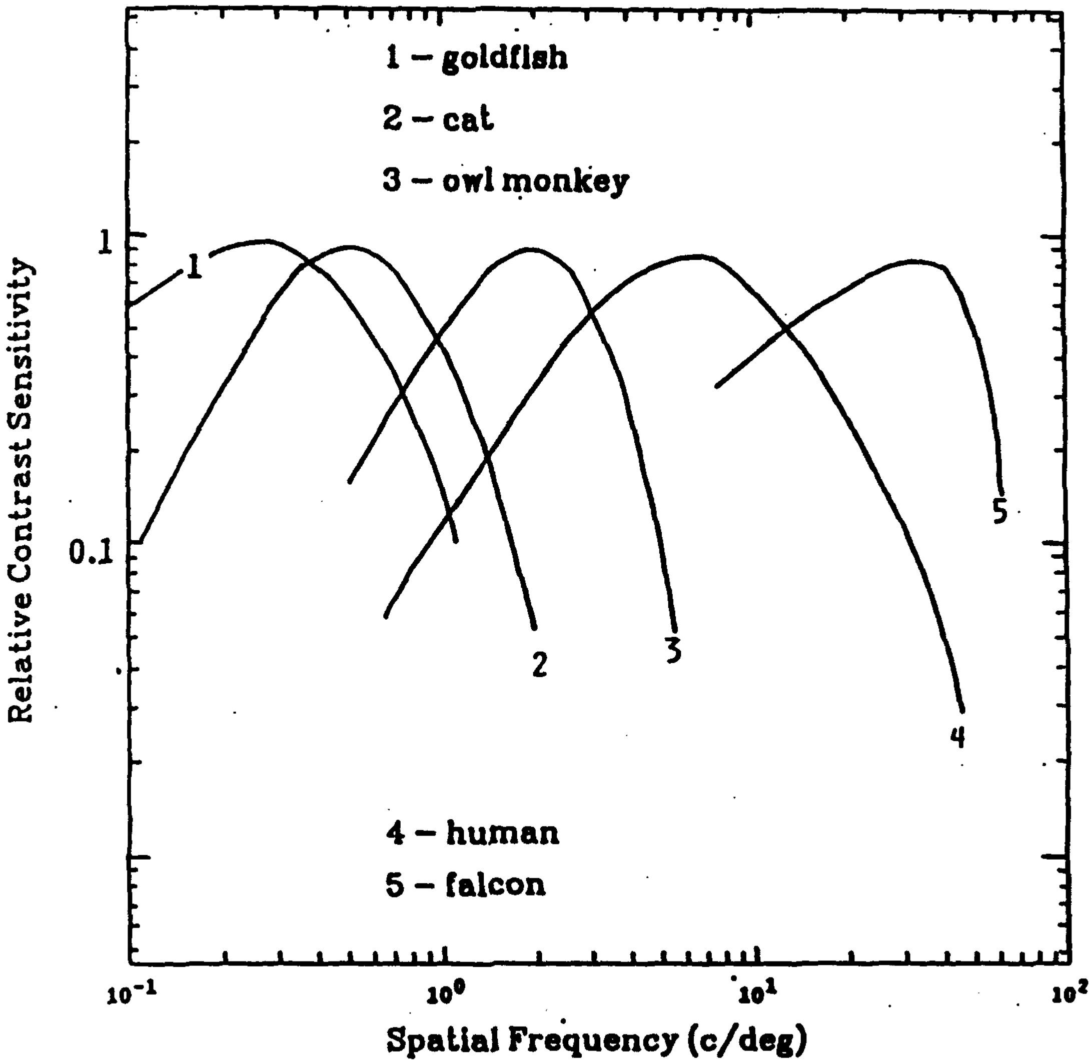

Figure 5.3. Contrast sensitivity function of the eyes of various species. (After DeValois and DeValois.[8])

a certain distance from the tank, the bulb became sharper when I moved closer to the tank. When I was closer, more details of the image of the light bulb were larger in angular width than the angular width corresponding to the highest spatial frequency at which the contrast exceeded the contrast threshold.

Different species have eyes with different contrast sensitivity functions[8] (see Fig. 5.3). A falcon, whose eye is sensitive not only to slightly higher spatial frequencies than a human's eye, but also to lower minimum levels of contrast, sees a sharper sun than a human does. Since the smallest size a cat can detect is about 0.3° in angular width, a cat would see the sun, but it would appear fuzzier to a cat than to a human. The cause of the fuzzy sun may be moot to even an extremely intelligent goldfish. A goldfish may never see the sun at all because the smallest size a goldfish can detect is about 1° in angular width. A remark by David Lynch at the 1993 Optical Society of America Topical Meeting on Light and Color in the Open Air that he had never seen a fuzzy sun while using a telescope to look at the sun through clouds is an indication of the relevance of the angular width of the details of an object to image resolution. The telescope allows higher spatial frequencies to be sampled and causes a sharper image of the sun to be seen.

Chapter 6

THE CLOUD: AN INTERVENING MEDIUM

A cloud, as an intervening medium between an object and the eye, reduces contrast between the image of the object and the image of its background. A cloud degrades image resolution when contrast reduction increases with spatial frequency. The modulation contrast function, which decreases as spatial frequency increases, represents the spatial frequency dependence of the reduction in contrast between the image of an object and the image of its background due to an intervening medium.[10]

Much of what I have observed while looking at the sun through clouds and during the experiment can be explained by the following hypothesis: The rate of decrease of the modulation contrast function with increasing spatial frequency becomes greater as the diameter of the scattering particles increases. Figure 6.1 is a schematic illustration of this hypothesis. Notice that if a cloud were composed of relatively small particles, the decrease in its modulation contrast function as spatial frequency increases is gradual (MCF_{1a} and MCF_{1b}). A certain increase in the optical thickness of such clouds, due either to an increase in

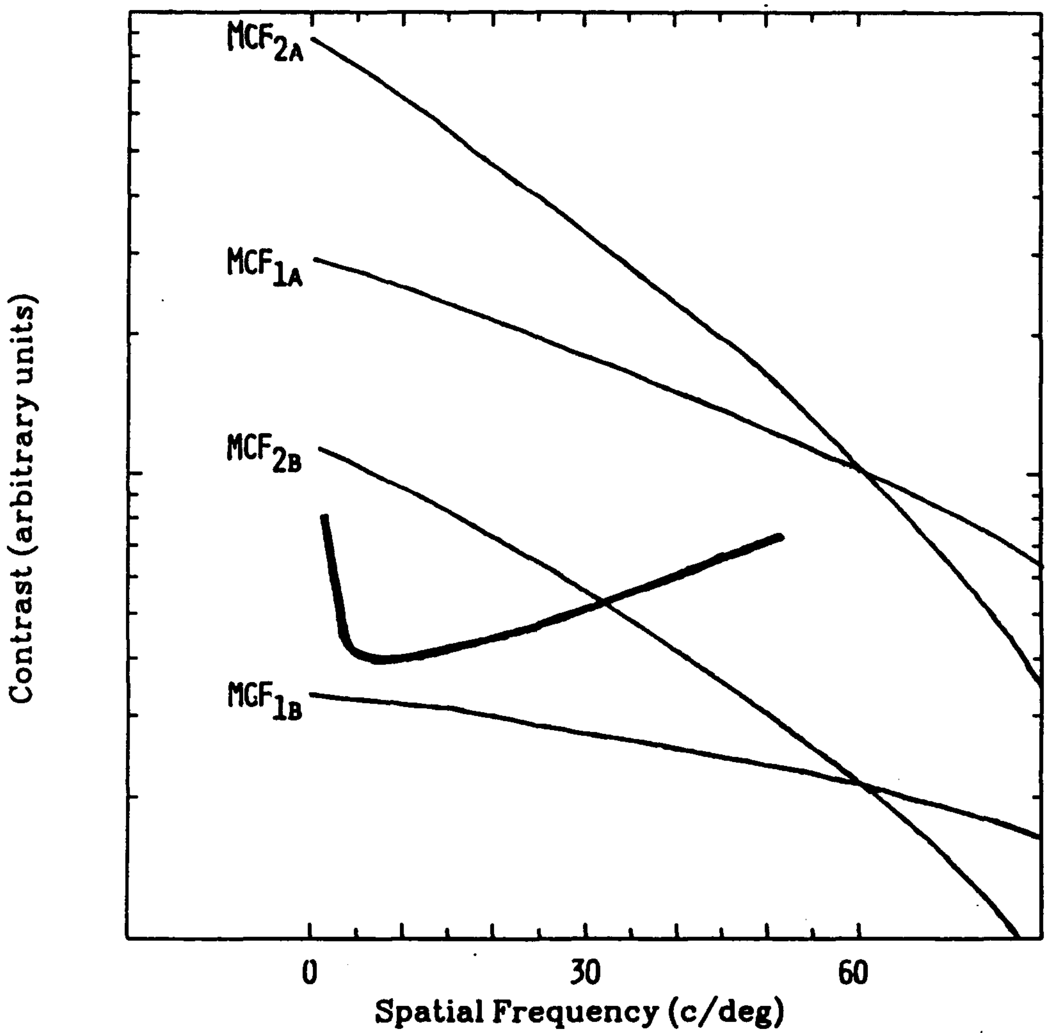

Figure 6.1. Schematic illustration that the rate of decrease of the modulation contrast function as spatial frequency increases becomes greater as the diameter of the particles increases. The dark line is the inverse of the eye's contrast sensitivity function. MCF_1 is for smaller particles and MCF_2 is for larger particles. Notice that the range of optical thicknesses at which higher spatial frequencies are lost but lower spatial frequencies are retained is greater with larger particles.

physical thickness or to an increase in scattering coefficient, causes the modulation contrast function to change from being greater than the threshold contrast at all spatial frequencies (MCF_{1a}) to being less than the threshold contrast at all spatial frequencies (MCF_{1b}). The sun seen through such clouds would change relatively quickly as optical thickness increases from being sharp-edged to not being visible at all. Conversely, if a cloud were composed of relatively large particles, the decrease in its modulation contrast function as spatial frequency increases would be more pronounced (MCF_{2a} and MCF_{2b}). The same increase in optical thickness causes the modulation contrast function to change from being greater than the contrast threshold at all spatial frequencies (MCF_{2a}) to being less than the threshold contrast at only the higher spatial frequencies at which the eye is sensitive (MCF_{2b}). The modulation contrast function is still greater than the threshold contrast at lower spatial frequencies. The sun seen through such clouds would change relatively slowly as optical thickness increased from being sharp-edged to not being visible. A greater change in optical thickness would be necessary for the sun to change from sharp-edged to fuzzy to not visible at all. This hypothesis is partially supported by theory; it is supported by observations of the sun through clouds and experimental results on three occasions where the theory is inconclusive.

Clouds reduce contrast by decreasing the luminance of the image of the sun and increasing the background luminance. The spatial frequency dependence of each is represented by a modulation transfer function (MTF), which is the spatial frequency dependent transmission of a signal through a medium.[11]

The spatial frequency dependence of the transmission of the image of an object through an intervening cloud is given by its modulation transfer function due to particles (M_s), which Lutomirski[12] presented as

$$M_s = \exp(-\sigma_s z \pi^2 f^2 / 3\alpha^2), \qquad (6.1)$$

where σ_s is the scattering coefficient of the cloud particles, z is the physical path length (the distance through the cloud along the observer's line of sight), f is the spatial frequency, and α is a parameter proportional to the effective mean diameter of the cloud particles. The product of σ_s and z defines the optical thickness of the cloud. α is obtained by a least-squares fit to the forward peak of the Henyey-Greenstein phase function.[13] The relationship between the diameter of a cloud particle, its asymmetry parameter, and the least-squares fit of α to the forward peak of its Henyey-Greenstein phase function can be seen in Table 6.1, which is for non-absorbing spheres at visible wavelengths. Equation (6.1) is valid at frequencies less than a cutoff frequency,

f_c, which is related to the diameter of the cloud particles, the focal length of the sensor, and wavelength. Zardecki et al.[13] and Lutomirski[12] each presented a method to compute f_c. Modulation transfer functions depend on spatial frequency below the cutoff frequency; they are independent of spatial frequency above the cutoff frequency. Observations of the sun through clouds and the results of the experiment indicate that when the sun is viewed through clouds, the spatial frequencies at which the human eye is sensitive are lower than the cutoff frequency. If this were not true, fuzziness would not have been apparent during the experiment or when the sun was viewed through clouds.

Table 6.1. The relationship between the diameter of a cloud particle and a least squares fit to the forward peak of its Henyey-Greenstein phase function.

$D_{spheres}$ (μm)	g	α (rad^{-1})
0.7	0.6	2.3
3.6	0.8	4.7
29.2	0.9	7.6

Note: Values were obtained by using the relationship between g and α determined by Zardecki et al.,[13] and computing the relationship between the diameter of the spheres and the asymmetry parameter using equations and tables published in *Multiple Light Scattering.*[22]

Equation (6.1) indicates that the modulation transfer function of the image of the object due to cloud particles decreases as the spatial frequency increases. Also, taking the derivative of Eq. (6.1) with respect to spatial frequency, and taking the derivative of that function with respect to α, shows that the rate of decrease of the modulation transfer function of the image due to cloud particles increases as the particle diameter increases:

$$\frac{\partial^2 M_s(f)/\partial f}{\partial\alpha} = \frac{4\sigma_s z\pi^2 f}{3\alpha^3}\left(1 - \frac{\sigma_s z\pi^2 f^2}{3\alpha^3}\right)\exp\left(\frac{-\sigma_s z\pi 2f}{3\alpha^2}\right). \quad (6.2)$$

Equation (6.2) is negative because the second term inside the first set of brackets is much greater than unity, and all other terms are positive. The sign of the second term inside the first parentheses can be determined using the following characteristic values: f, 30 c/deg; z, 100 m; α, 0.082 c/deg (4.7 c/rad); σ_s, 5×10^{-2} m^{-1}. Equation (6.1) indicates that the modulation transfer function due to particles decreases when an increase in the scattering coefficient or an increase in the physical path length causes the optical thickness to increase.

That the modulation transfer function of the image of the sun due to cloud particles decreases as spatial frequency increases and that the rate of decrease increases as the diameter of the cloud particles increases is not sufficient

for the modulation contrast function to behave as predicted by the hypothesis. The modulation transfer function of the background luminance must change in a way that, together with the modulation transfer function of the image of the sun due to cloud particles, causes the modulation contrast function to behave as predicted by the hypothesis.

The modulation transfer function that Kopeika[10] presented for background radiance has been modified for application to the appearance of the sun through clouds. Absorption has been ignored because it is negligible for water at visible wavelengths. Wavelength dependence has been ignored because only the visible wavelength is of interest. The modified modulation transfer function for background luminance is

$$M_B \propto \frac{1}{\sigma_s}\left[1 - \exp(-1) + \left(\frac{f_c}{f}\right)^2 \left(\exp\left(\frac{f}{f_c}\right)^2 - \exp\left(-\left(\frac{f}{f_c}\right)^2 \sigma_s z\right)\right)\right]. \quad (6.3)$$

Equation (6.3) increases with spatial frequency because the second exponential term is greater than the third exponential term, and the second exponential term increases as spatial frequency increases while the third exponential term decreases as spatial frequency increases. Also, the results of an experiment performed by Kopeika et al.[11] have confirmed that background luminance increases as spatial frequency increases.

Equation (6.3) indicates that background luminance increases as the optical thickness of the intervening medium increases.

The derivative of Eq. (6.3) with respect to spatial frequency is

$$\frac{\partial M_B}{\partial f} \propto \frac{2}{\sigma_s f}\left[1-\left(\frac{f_c}{f}\right)^2\right]\exp\left(\frac{f}{f_c}\right)^2+\frac{2}{f}\left[z+\frac{1}{\sigma_s}\left(\frac{f_c}{f}\right)^2\right]\exp\left(-\left(\frac{f}{f_c}\right)^2\sigma_s z\right). \quad (6.4)$$

It is not apparent from Eq. (6.4) how an increase in particle diameter alters the increase in background luminance as a function of spatial frequency. When σ_s increases due to an increase in particle diameter,[14] the first term, which is negative because f_c/f is greater than unity, becomes less negative but the second term becomes less positive. Also, it is not clear how an increase in f_c, which is proportional to particle diameter, alters the increase in background luminance with spatial frequency. Based on the results of his experiment, Kopeika[11] concludes that the increase in background luminance with spatial frequency becomes less pronounced as particle diameter increases.

Increases in particle diameter enhance the decrease in the modulation transfer function of the image of the sun due to cloud droplets as spatial frequency increases, but retard the increase in the modulation transfer function of background luminance as spatial frequency increases. Based on the

results of the experiment, and on my observations of the sun through clouds, the magnitude of the first change is expected to be greater than the magnitude of the second change. The decrease in the modulation contrast function as spatial frequency increases becomes greater when the particle diameter increases. The range of optical thicknesses over which fuzzy sun are observed increases. That Eq. (6.1) decreases and Eq. (6.3) increases as optical thickness increases is expected because contrast between the sun and an intervening cloud is observed to decrease as the intervening cloud becomes optically thicker.

Turbulence and molecular scattering in an intervening medium also degrade image resolution, but they are not necessary for fuzzy suns to be seen through clouds. Significant turbulence is not present when the sun is seen through an overhead cloud. Molecular scattering is negligible compared with scattering by cloud particles at visible wavelengths.[11]

Chapter 7

CLOUD PROPERTIES

A cloud must have certain properties for a fuzzy sun to be seen through it. First, it must be composed of cloud particles that are relatively large. Fuzzy light bulbs were observed in the laboratory through media composed of small particles, but the range of optical thicknesses associated with the fuzziness was small and careful observation was necessary to detect the fuzzy light bulbs through the small particles. Outside the laboratory, fuzzy suns will be more noticeable through clouds of larger particles. Such clouds have a greater range of optical thicknesses for which fuzziness is possible, and the contrast between the sun (or light bulb) and the background at which the fuzziness is seen is greater. Second, the cloud can be neither too thin nor too thick. If it is too thin, the sharp edge of the disk will be distinct from the aureole. If it is too thick, the sun will be obscured.

The properties of clouds, such as particle size distribution, number density of particles, and optical thickness are highly variable, so making precise statements about the properties of cloud types should be avoided.

However, there are indications that altostratus is more likely than other cloud types to be characterized by a combination of large particles and moderate optical thickness necessary for fuzzy suns to be seen through it.

In the *International Cloud Atlas*, altostratus, along with nimbostratus and stratocumulus, is described as having a layer of raindrops as part of its structure. Quite frequently, altostratus also has a layer of a mixture of ice crystals, snow crystals, and snowflakes.[2] A layer of raindrops or a layer of crystals in a cloud will increase the mean diameter of the cloud particles. A typical cloud droplet has a radius of about 10 μm, but a typical raindrop has a radius of about 1000 μm. Even the smallest raindrops, associated with non-precipitating clouds, have a mean radius of about 100 μm.[15] Ice crystals, snow crystals, and snowflakes also are larger than cloud droplets.[16] That fuzzy suns are most commonly observed in the winter may be evidence that the presence of ice crystals, snow crystals, or snowflakes (because of their size, not because of their shape) increase the mean particle diameter of a cloud. A layer of raindrops or a layer of crystals, even if their number density is only a few percent of the number density of cloud droplets, will appreciably alter the particle spectrum and increase the mean diameter of the cloud particles.

Continental radiation fog, the cloud through which I have observed many remarkable sharp-edged suns, is composed of

smaller droplets than other clouds. Two factors keep the size of fog droplets small. There is an abundance of cloud condensation nuclei, so for a given liquid water content many small droplets form. Also, vertical motion in fog is extremely weak so the droplets do not grow by collision and coalescence. The mean diameter of radiation fog is typically between about 6 μm and 12 μm, which is smaller than the mean diameter of the droplets in most clouds.[17, 18]

Stratus, the other cloud through which frequently I have observed sharp-edged suns, is also composed of smaller than average droplets. Vertical air motion in stratus clouds is weaker than in cumulus clouds. Therefore, the droplets in stratus, like those in radiation fog, grow by condensation rather than by coalescence, which keeps the mean diameter small and the droplet spectrum narrow.[15] Stratus cloud droplets have been found to have mean diameters of about 10 μm, which is smaller than the mean diameter of the droplets in most clouds.[19]

During the fleeting moments that I have been able to see the sun through stratocumulus or weak cumulus, the sun was visible only through the edges of the cloud, and the sun had a sharp edge. The sun always has been obscured by the optically thicker sections of these clouds. Due to the entrainment of dry air, the edges of clouds are more tenuous and composed of smaller droplets than the interior section of clouds are.[15] These are the conditions that favor the

appearance of the sharp-edged sun. Away from the edges of the cloud, the droplets are larger, but the optical depth is also greater, and the sun is obscured.

High clouds, such as cirrus and cirrostratus, are composed predominantly of ice crystals, which are among the larger particles in clouds. But cirrus is so thin that I have never been able to observe the sun through cirrus without using sunglasses or looking at the reflection of the sun to reduce the luminance. With the luminance reduced sufficiently to attenuate the aureole, I always have seen a sharp-edged sun. As mentioned previously, it can be difficult to distinguish one cloud type from another; it is especially easy to confuse altostratus with cirrostratus. But the *International Cloud Atlas* indicates that cirrostratus is optically thinner than altostratus.[2] I have not observed fuzzy suns through cirrostratus because it is too optically thin.

For a fuzzy sun to be seen, the cloud through which it is viewed must not only be composed of large particles, but must also be of a certain optical thickness: neither too great nor too small. Precipitation is associated with nimbostratus but not altostratus, so the raindrops in nimbostratus are larger than the raindrops in altostratus.[20] The mean diameter of the drops in nimbostratus is large enough to cause a fuzzy sun. But nimbostratus, which is optically thicker than altostratus, is too optically thick for the sun to be observed.[2, 21] The

same is true for vigorous cumulus and stratocumulus away from their edges. With careful, persistent observation, it might be possible to view a fuzzy sun through clouds other than altostratus. But altostratus is the cloud most likely to be composed of sufficiently large particles, and to be of the proper optical thickness for a fuzzy sun to be seen through it.

Chapter 8

CONCLUSION

Fuzzy suns can sometimes be seen through altostratus because altostratus is the cloud most likely to be composed of sufficiently large cloud particles, yet be of moderate optical thickness. The large mean particle diameter is caused by layers of raindrops and layers of crystals that are often in altostratus. Other clouds can be composed of large particles also, but they are typically too optically thin to cause a fuzzy sun, or too optically thick for the sun to be seen at all.

Large particles increase the rate at which the modulation contrast function of the cloud decreases as spatial frequency increases. The steeper decrease in the modulation contrast function causes the transition between the modulation contrast function's being greater than the contrast sensitivity function of the eye at all spatial frequencies to its being less than the contrast sensitivity function of the eye at all spatial frequencies to be more gradual. Therefore, the range of optical thicknesses at which lower spatial frequencies but not higher spatial frequencies are retained is greater and the fuzzy image is more noticeable. It may be possible to

observe a fuzzy sun through a cloud that is not altostratus, but I have never done so yet.

Appendix

THE APPLICATION OF MONTE CARLO TECHNIQUES TO PROBLEMS OF IMAGE RESOLUTION

Monte Carlo techniques are often used with good results to simulate radiative transfer when the equation of radiative transfer cannot be solved. Fundamentally, Monte Carlo simulation is the application of probability to determine an outcome. Because each outcome is decided randomly, it is not meaningful individually. But with enough outcomes, a pattern can be detected. The generally used example is tossing a coin. The outcome of one toss of a coin will be heads or tails. The outcome of two, three, or four tosses of a coin might very well be a string of heads or a string of tails. But if the coin is tossed enough times, a pattern will be detected. In general, half the time a coin is tossed the outcome will be heads, and half the time the outcome will be tails.[23,24]

I attempted to model the appearance of the sun through clouds using Monte Carlo techniques, but with less than complete success. The transmittance and reflectance of sunlight by a cloud were modelled quite well, but the appearance of the sun through clouds was not. The success of

a proper Monte Carlo simulation depends ultimately on two things: adequate computing power and an adequate random number generator. Adequacy is emphasized because a random number generator can be suitable for a simple application, but not for a more advanced one. Similarly, a computer that can run a model of a simple phenomenon in a few minutes may take weeks or even months to run a model of a more subtle phenomenon. This appendix contains a description of the Monte Carlo model I developed. I refer to modelled photons, modelled clouds, and modelled cloud particles without the important adjective modelled. I have done this for ease of reading; I hope that in so doing, I have not contributed to confusing a model with what it models. Lines of computer code, however cleverly written, will never be the atmosphere.

The cloud is modelled as a homogeneous medium that is infinite in lateral extent. A photon may be scattered out the bottom or out the top of the cloud, but it cannot be scattered out the side. Not all clouds can be reasonably approximated as infinite in lateral extent, but fog, stratus, and altostratus can be. Fog and stratus are associated with sharp-edged suns; altostratus is associated with fuzzy suns. These clouds are not always horizon-to-horizon in extent, but they generally cover a large surface area. The asymmetry parameter, g, and the optical thickness can be specified for the modelled cloud.

Photons enter the top of the cloud at a randomly determined angle from the reference vector (a downward pointing vector which is orthogonal to the top of the cloud; the reference vector is parallel to the z-axis) shown in Figure A.1. Photons travel a randomly determined distance before striking a cloud particle. The distance traveled before striking a cloud particle is determined using the following equation:

$$d = -\log(\mu), \qquad \text{(A.1)}$$

where μ is a number between 0 and 1 that is selected randomly from a uniform distribution, and d is the distance (in optical thickness) traveled by the photon before striking a cloud particle. Therefore, a photon may travel any distance before striking a cloud particle, but most of the distances are relatively short.

The vertical distance travelled is then compared with the depth of the cloud. If the photon passed through the cloud and out the bottom, the angle of its path relative to the reference vector is retained for binning. Binning will be described later.

If the photon did not travel out the bottom of the cloud, scattering is simulated. Absorption is not modelled because it is insignificant compared with scattering of visible light by cloud droplets. Two scattering angles relative to those

droplets and visible wavelengths. Two scattering angles relative to those just computed are randomly determined. See Figure A.1. φ is the angle relative to the path that the photon had just taken. Zero degrees is a continuation of the previous path. One hundred and eighty degrees is a reversal of the previous path. φ is determined randomly using the asymmetry parameter determined from the Henyey-Greenstein

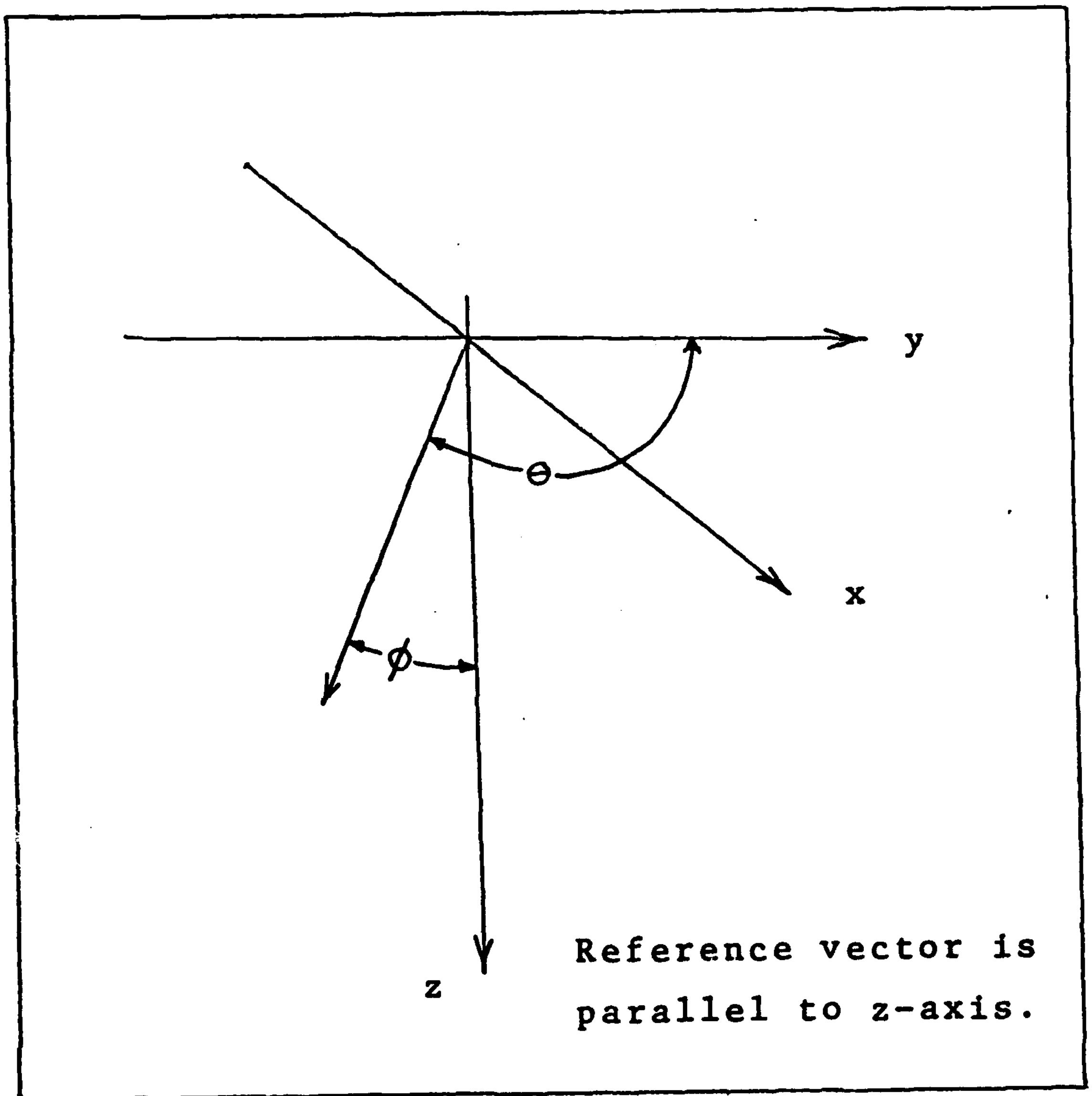

Figure A.1. Illustration of the reference vector, φ, and θ.

previous θ. It is drawn randomly from a uniform distribution of angles between 0° and 360°. The corresponding angles relative to the reference vector are then computed, and the new position of the photon is determined.

The process described in the preceding two paragraphs is repeated until the photon goes out the bottom of the cloud or goes out the top of the cloud. If it goes out the bottom of the cloud, φ is retained for binning.

Many photons are simulated and the following statistics are obtained: the percentage of photons reflected out the top of the cloud, the percentage of photons transmitted through the bottom of the cloud, and the number of photons that went out the bottom of the cloud at certain ranges of angles called bins. The bins are a set of concentric rings and the angular area for each bin is the same. The angular distance between the inner and outer edge of each ring becomes smaller as the distance from each ring to the center becomes greater. Therefore, the area of the rings is constant and the number of photons falling into each ring is proportional to the radiance at each ring's angular distance from the center. Thirty bins are used; the first fifteen are within the angular width of the sun; the last fifteen are outside the angular width of the sun.

The number of bins was selected so the angular width of the bin near the edge of the sun was approximately the same as the smallest angular width that can be seen by the human eye.

This angular width was determined by asking colleagues to watch as I moved two sheets of black paper relative to one another with a white sheet of paper as a background. Each colleague told me when the top edge of one sheet of black paper appeared to be even with the top edge of the other sheet. The linear distance was converted to an angular distance and the mean angular distance, 0.0167 radians, was used as the angular width of the fifteenth bin.

The results of the model provided estimates for transmittance and reflectance for clouds of a certain optical thickness and asymmetry parameter that agree to four significant digits with those reported by van de Hulst in *Multiple Light Scattering.*[25] The model results concerning image resolution were inconclusive, but could be improved with a faster computer. For example, over 200 million photons were simulated through a cloud with an optical thickness of 15. Of those 200 million, only 5019 were binned, with about 180 photons going into each bin. With so few photons in each bin, random fluctuations masked any systematic differences between bins. Since the difference between two adjacent bins must be less than about 2% before one bin can be distinguished from the other by the eye, randomness must be minimized. Randomness can be minimized by increasing the number of photons modelled, but simulating 200 million photons took about a week using even the Cray computer (at a relatively low priority) at Penn State. Even with enough photons, the

adequacy of the random number generator, especially at the least significant digits, must be doubted when the random number generator is used so much. At an optical depth of 15, about 100 random numbers are used for each photon to exit the cloud.

A copy of the computer code begins on the next page.

```
      Program Main

      logical*4 cont
      character*11 sctrg
      character*3 yes
      integer count,numoutbtm,numphtns,nvar
      real*8 pi,twopi,edge,tau,g,aa,bb,cc,dd,ee,r,m1,m2,sctrphi,
     &   x
      parameter (pi = 3.141592654, twopi = 6.283185308,
     &   tau = 3.9, g = 0.85)
      parameter (nvar = 31)

      dimension count(40), edge(31), r(3,1), m1(3,3), m2(3,3),
     &    x(3,1)
      common cont

      print *, "Type 'yes'"
      read *, yes

      open (unit=5,file="b:nrw39.g85",status="unknown")
      rndnumgeninitialization = RRAND()

      call BinEdge(edge,twopi)
      call DefineCloud(g,aa,bb,cc,dd,ee)

      cont = .false.
      do while (cont .eqv. .false.)
         option break (cont)
         call GeneratePhoton(r,m1,m2,sctrphi,twopi,numphtns)
         call CheckforPassage(r,tau,numoutbtm,sctrg)

         do while (sctrg .eq. 'continue')
           call Scatter(r,m1,m2,sctrphi,g,aa,bb,cc,dd,ee,x,twopi)
           call CheckforPassage(r,tau,numoutbtm,sctrg)
         end do

         if (sctrg .eq. 'complete') then
            call BinPlace(sctrphi,count,edge,nvar)
         end if
      end do

      call SendtoFile(count,numphtns,tau,numoutbtm,g)
      close (5)

      stop
      end

c******Subroutine Bin Edge*******************************************
c
c  Computes the edges of the bins.
c  For horizon to horizon, area = 0.20943952
c  For 2x radius of the sun, area = 4.31117291555d-06
c
c*******************************************************************

      Subroutine BinEdge (edge,twopi)

      real*8 twopi, cosedge, area, edge
      dimension cosedge(31), edge(31)

      area = 4.3111729155d-06
      cosedge(1) = 1.0
      do n = 2, 31
        nminus1 = n - 1
        cosedge(n) = -(area/twopi) + cosedge(nminus1)
        edge(n) = DACOS(cosedge(n))
      end do
      edge(1) = 0.0
      return
      end
```

```
c******Subroutine Bin Place******************************************
c
c       Places photons that pass through the bottom of the cloud
c       into a bin, based on the scattering angle of the photon.
c
c********************************************************************

        Subroutine BinPlace (sctrphi,count,edge,nvar)

        real*8 sctrphi, edge
        integer count, cnvrsnphi, nvar
        dimension count(40), edge(nvar)

        if (sctrphi .lt. edge(31)) then
         do i = 1, 30
           iplus1 = i + 1
           if (sctrphi.ge.edge(i).and.sctrphi.lt.edge(iplus1)) then
              cnvrsnphi = i
           end if
         end do
         count(cnvrsnphi) = count(cnvrsnphi) + 1
        end if
        return
        end

c******Subroutine Check for Passage*********************************
c
c       Checks whether the photon has passed out the top or
c       bottom of the cloud.
c
c********************************************************************

        Subroutine CheckforPassage (r,tau,numoutbtm,sctrg)

        real*8 r, tau
        integer numoutbtm
        character*11 sctrg

        dimension r(3,1)

        sctrg = 'continue'
        if (r(3,1) .gt. tau) then
           sctrg = 'complete'
           numoutbtm =  numoutbtm + 1
        else if (r(3,1) .lt. 0.0) then
           sctrg = 'sctrdouttop'
        end if
        return
        end

c******Subroutine Define Cloud***************************************
c
c       Defines the optical depth of the cloud and the assymetry
c       parameter of the cloud.  (aa to ee defined here for speed
c       in Scatter.
c
c********************************************************************

        Subroutine DefineCloud (g, aa, bb, cc, dd, ee)

        real*8 g, aa, bb, cc, dd, ee

        aa = 1 - g
        bb = 2 * g
        if (g .ne. 0.0) then
           cc = 1/(2*g)
        end if
        dd = 1 + g**2
        ee = 1 - g**2
        return
        end
```

```
c******Subroutine Generate Photon*********************************
c
c      Generates a photon.
c
c*****************************************************************

       Subroutine GeneratePhoton(r,m1,m2,sctrphi,twopi,numphtns)

       integer numphtns
       real*8 r, m1, m2, sctrphi, freepath, t, s, c, u, v, sctrthta,
     &   cossctrphi, twopi
       dimension r(3,1), m1(3,3), m2(3,3)

       numphtns = numphtns + 1

       freepath = -DLOG(RND() + 0.000000001)
       t= freepath

       sctrthta = twopi * RND()
       s = DSIN(sctrthta)
       c = DCOS(sctrthta)

       cossctrphi = 1.01
       do while (cossctrphi .ge. 1.0)
          cossctrphi = 0.999989725 + 0.000010274 * RND()
       end do
       sctrphi = DACOS(cossctrphi)

       u = DCOS(sctrphi)
       v = DSIN(sctrphi)

       r(1,1) = v*c*t
       r(2,1) = v*s*t
       r(3,1) = u*t

       m1(1,1) = 1.0
       m1(1,2) = 0.0

       m1(1,3) = 0.0
       m1(2,1) = 0.0
       m1(2,2) = 1.0
       m1(2,3) = 0.0
       m1(3,1) = 0.0
       m1(3,2) = 0.0
       m1(3,3) = 1.0

       m2(1,1) = u*c
       m2(1,2) = -s
       m2(1,3) = v*c
       m2(2,1) = u*s
       m2(2,2) = c
       m2(2,3) = v*s
       m2(3,1) = -v
       m2(3,2) = 0.0
       m2(3,3) = u

       return
       end
```

```
c******Subroutine Scatter*****************************************
c
c       Scatters photon.
c
c****************************************************************

        Subroutine Scatter (r,m1,m2,sctrphi,g,aa,bb,cc,dd,ee,x,twopi)

        real*8 r,m1,m2,sctrphi,g,aa,bb,cc,dd,ee,x,x0,m3,twopi,
     &    cossctrphi,summation,t,u,v,c,s,sctrthta
        dimension r(3,1),m1(3,3),m2(3,3),m3(3,3),x(3,1),x0(3,1)

        sctrthta = twopi * RND()
        s = DSIN(sctrthta)
        c = DCOS(sctrthta)

        if (g .eq. 0.0) then
           cossctrphi = 2 * RND() - 1
        else
           summation = 0.0
           do while (summation .eq. 0.0)
              summation = aa + bb * RND()
           end do
           cossctrphi = cc * (dd - (ee / summation)**2)
        end if

        sctrphi = DACOS(cossctrphi)

        u = cossctrphi
        v = DSIN(sctrphi)

        t = -DLOG(RND() + 0.0000001)

        x(1,1) = v*c*t
        x(2,1) = v*s*t
        x(3,1) = u*t

        do i = 1, 3
           do j = 1, 3

              m3(i,j) = m1(i,1)*m2(1,j) + m1(i,2)*m2(2,j) +
     &                  m1(i,3)*m2(3,j)
           end do
        end do

        do i = 1, 3
           x0(i,1) = x(i,1)
        end do

        do i = 1, 3
           x(i,1) = m3(i,1)*x0(1,1)+m3(i,2)*x0(2,1)+m3(i,3)*x0(3,1)
        end do

        do i = 1, 3
           r(i,1) = r(i,1) + x(i, 1)
        end do

        sctrphi = DATAN((((x(1,1))**2+(x(2,1))**2)**0.5)/x(3,1))

        m2(1,1) = u*c
        m2(1,2) = -s
        m2(1,3) = v*c
        m2(2,1) = u*s
        m2(2,2) = c
        m2(2,3) = v*s
        m2(3,1) = -v
        m2(3,2) = 0.0
        m2(3,3) = u

        do i = 1, 3
           do j = 1, 3
              m1(i,j) =m3(i,j)
           end do
        end do

        return
        end
```

```
c******Subroutine Send to File************************************
c
c      Sends results to file.
c
c******************************************************************

       Subroutine SendtoFile(count,numphtns,tau,numoutbtm,g)


       real*8 tau, g
       integer count, numphtns, numoutbtm, numouttop, numbinned
       dimension count(31)

       numbinned = 0
       numouttop = numphtns - numoutbtm

       do n = 1, 30
          write (5,*) count(n)
          numbinned = numbinned + count(n)
       end do
       write (5,*) numphtns
       write (5,*) numoutbtm
       write (5,*) numouttop
       write (5,*) numbinned
       write (5,*) tau
       write (5,*) g
       write (5,*) "number of photons"
       write (5,*) "number scattered out bottom"
       write (5,*) "number scattered out top"
       write (5,*) "number binned"
       write (5,*) "tau"
       write (5,*) "asymmetry parameter"
       return
       end
```

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